Algebraic Geometry I
Winter term 2021/2022 Florian Strunk

Exercise sheet 06

To be handed in until Wednesday, 01st of December, 4pm

Exercise 1 (4 Points)

1. Show the assertion in Example 8.4.(2), i.e. show that for $f\colon \{y\} \hookrightarrow Y$ the stalk of the *skyscraper sheaf* $f_* acA$ at a point $y' \in Y$ is
$$(f_* acA)_{y'} \cong \begin{cases} A & \text{if } y' \in \overline{\{y\}}, \\ * & \text{else.} \end{cases}$$

2. Show the assertion in Example 8.4.(3), i.e. show that for a closed subspace $f\colon Z \hookrightarrow X$ the stalk of the *extension by zero sheaf* $f_*\mathcal{F}$ at a point $y \in Y$ is
$$(f_*\mathcal{F})_y \cong \begin{cases} \mathcal{F}_y & \text{if } y \in Z, \\ * & \text{else.} \end{cases}$$

Exercise 2 (4 Points) Let X be a topological space and F_x a set or every $x \in X$. We have shown that the assertion
$$\begin{array}{rccl} \prod_{x\in(-)} F_x\colon & \mathbf{Ouv}(X)^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \prod_{x\in U} F_x \end{array}$$
is a sheaf (it is not needed that the F_x arise as the stalks of a presheaf). Show or disprove: The stalk of this sheaf at x is F_x.

Exercise 3 (4 Points) Show Lemma 8.13. of the lecture, i.e. show that the assertion
$$\begin{array}{rccl} \underline{\mathrm{Hom}}(\mathcal{F},\mathcal{G})\colon & \mathbf{Ouv}(X)^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \mathrm{Hom}_{\mathbf{PSh}(U)}(\mathcal{F}_{|U},\mathcal{G}_{|U}) \end{array}$$
is a sheaf for a presheaf $\mathcal{F}$ and a sheaf $\mathcal{G}$ on a topological space X.

Exercise 4 (4 Points) In this exercise, we want to glue topological spaces. Consider the diagram
$$\begin{tikzcd} \{0,1\} \arrow[r, hook, "i"] \arrow[d, hook, "j"'] & {[0,1]} \arrow[d, hook] \\ {[0,1]} \arrow[r, hook] & P \end{tikzcd}$$
of topological spaces where $[0,1]$ denotes the unit interval, $\{0,1\}$ the set of the two endpoints and $i = j$ the obvious inclusions. Show that the glued space P (as definied in the lecture) is homeomorphic to the unit circle $S^1 := \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$.

(Hint: You may use the fact that a bijective continuous map from a quasi-compact to a Hausdorff-space is a homeomorphism.)