Exercise sheet 05

To be handed in until Wednesday, 24th of November, 4pm

Exercise 1 (4 Points) Let X be a topological space, $\mathcal{F}: \mathbf{Ouv}(X)^{op} \to \mathbf{Set}$ a sheaf and let $s, t \in \mathcal{F}(U)$ be two sections for some open $U \subseteq X$.

1. Show that $\{x \in U \mid s_x = t_x\}$ is an open subset of U.
2. Show that $s = t \iff \forall x \in U : s_x = t_x$.

Exercise 2 (4 Points) Consider the presheaf

$$\begin{array}{rccl} \mathcal{C}_{bd}(-,\mathbb{R}): & \mathbf{Ouv}(\mathbb{R})^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \{f: U \to \mathbb{R} \mid f \text{ is continuous and bounded}\} \end{array}$$

and show that it is not a sheaf.

Exercise 3 (4 Points) Consider the circle $S^1 \subseteq \mathbb{R}^2$, two points $p, q \in S^1$ with $p \neq q$ and the presheaf

$$\begin{array}{rccl} \mathcal{F}: & \mathbf{Ouv}(S^1)^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \{f \in \mathcal{C}(U,\mathbb{R}) \mid f(p) = f(q) \text{ if } p, q \in U\}. \end{array}$$

Show that it is not a sheaf and determine its sheafification.

Exercise 4 (4 Points) Let $X \subseteq \mathbb{C}$ be an open non-empty subset. Show that

$$\begin{array}{rccl} \mathcal{C}_{hol}(-,\mathbb{C}): & \mathbf{Ouv}(X)^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \{f \in \mathcal{C}(U,\mathbb{C}) \mid f \text{ is holomorphic}\}. \end{array}$$

is a sheaf. Show that the derivative $f \mapsto f'$ of a function $f: U \to \mathbb{C}$ induces a morphism $D: \mathcal{C}_{hol}(-,\mathbb{C}) \to \mathcal{C}_{hol}(-,\mathbb{C})$ of presheaves which is stalkwise surjective but not sectionswise surjective.