Florian Strunk

Exercise sheet 03

To be handed in until Wednesday, 10th of November, 4pm

Exercise 1 (4 Points) Let X be a topological space and $M \subseteq X$ a subset. Show that M is irreducible if and only if the closure $\overline{M}$ in X is irreducible.

Exercise 2 (4 Points) Let k be a field. Decompose the subspace

$$\mathcal{V}(x^2 - yz, xz - x)$$

of $\mathbb{A}^3_k = \mathrm{Spec}(k[x,y,z])$ into irreducible components.

(Hint: Show that each of them is isomorphic to $\mathbb{A}^1_k$.)

Exercise 3 (4 Points) Consider the image $Y \subseteq \mathbb{A}^3_{\mathbb{Q}}$ of the $\mathbb{Q}$-algebraic map

$$\begin{array}{rccc} F\colon & \mathbb{A}^1 & \to & \mathbb{A}^3 \\ & t & \mapsto & (t^3, t^4, t^5). \end{array}$$

More precisely, one considers the associated morphism

$$\begin{array}{rccl} \varphi\colon & \mathbb{Q}[x,y,z] & \to & \mathbb{Q}[t] \\ & x & \mapsto & t^3 \\ & y & \mapsto & t^4 \\ & z & \mapsto & t^5 \end{array}$$

of $\mathbb{Q}$-algebras and sets $Y := \mathrm{Spec}(\mathbb{Q}[x,y,z]/\ker(\varphi)) \cong \mathcal{V}(\ker(\varphi)) \subseteq \mathbb{A}^3$. Find generators of the ideal $\ker(\varphi)$ and show that Y is irreducible.

Exercise 4 (4 Points) Decompose the subspace

$$\mathcal{V}(xz - y^2, z^3 - x^5)$$

of $\mathbb{A}^3_{\mathbb{Q}} = \mathrm{Spec}(\mathbb{Q}[x,y,z])$ into irreducible components.

(Hint: Use Exercise 3.)