

Exercise sheet 20

Algebraic Geometry II
Summer term 2018

EXERCISE 1

Let A be a ring with a valuation $v : A \rightarrow \Gamma \cup \{\infty\}$. Show that $v^{-1}\{\infty\}$ is a prime ideal of A .

EXERCISE 2

Let Γ, \leq be a totally ordered abelian group. A nonempty subset $S \subset \Gamma_{\geq 0}$ is called an *ideal*, if for all $s \in S$ and we have

$$\{\gamma \in \Gamma \mid \gamma \geq s\} \subset S.$$

Let V be a valuation ring with a surjective valuation $v : V \rightarrow \Gamma_{\geq 0} \cup \{\infty\}$. Construct a bijection between ideals of V and ideals of Γ .

EXERCISE 3

Let V be a valuation ring with field of fractions K . Show that V is noetherian if and only if $K^\times/V^\times \cong \mathbb{Z}$.

EXERCISE 4

A scheme morphism $f : X \rightarrow Y$ is called an *immersion* if there is a factorization

$$f : X \xrightarrow{i} U \xrightarrow{j} Y$$

where i is a closed immersion and j is an open immersion.

1. Show that an immersion f is a closed immersion if and only if the image of f is closed in Y .
2. Give an example of an immersion with open image which is not an open immersion.
3. Let f be a quasi-compact immersion. Show that f can also be factored "the other way around", i.e. there is a factorization

$$f : X \xrightarrow{j'} Z \xrightarrow{i'} Y$$

with j' an open immersion and i' a closed immersion.