

Exercise sheet 21

Algebraic Geometry II
Summer term 2018

EXERCISE 1

An affine morphism $f : X \rightarrow Y$ is called *integral* if for all affine open $U \subset Y$, the induced map

$$\mathcal{O}_Y(U) \rightarrow \mathcal{O}_X(f^{-1}U)$$

is an integral map of rings. Show that integral morphisms are universally closed.

Hint: You may use that injective integral ring maps $A \rightarrow B$ induce surjections $\text{Spec}(B) \rightarrow \text{Spec}(A)$.

EXERCISE 2

Let k be a field. Show that $\mathbb{A}_k^1 \rightarrow \text{Spec}(k)$ is not proper.

EXERCISE 3

Find examples of diagrams

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ & \searrow h & \swarrow g \\ & & Z \end{array}$$

of morphisms of schemes with the following properties:

1. f and h are proper, g is separated and of finite type, but not universally closed.
2. h is proper, f is proper and surjective, g is not proper.
3. h is proper, f is a dominant open immersion which is not an isomorphism, and g is universally closed and of finite type.

EXERCISE 4

Show that the property of a scheme morphism to be surjective is stable under base change.