Exercise sheet 20

Algebraic Geometry II
Summer term 2018

Exercise 1

Let $f : X \to Y$ be a map of schemes. For the following list of properties of f, decide with proof resp. counterexample which implications hold:

1. f is finite.
2. f is of finite type.
3. f is affine.
4. f is proper.
5. f is quasi-compact.
6. f is integral.
7. f is separated.
8. f is integral and locally of finite type.
9. f is a closed immersion.
10. f is locally of finite type.
11. f is quasi-separated.
12. f is an open immersion.

Exercise 2

An affine morphism $f : X \to Y$ is called *integral* if for all affine open $U \subset Y$, the induced map

$$\mathcal{O}_Y(U) \to \mathcal{O}_X(f^{-1}U)$$

is an integral map of rings. Show that integral morphisms are universally closed.
Hint: You may use that injective integral ring maps $A \to B$ induce surjections $\mathrm{Spec}(B) \to \mathrm{Spec}(A)$.

Exercise 3

Let

$$\begin{array}{ccc} X & \xrightarrow{\quad f \quad} & Y \\ & {}_{h}\searrow \quad \swarrow_{g} & \\ & Z & \end{array}$$

be a commutative diagram of morphisms of schemes.

1. Suppose h is proper and g is separated. Show f is proper.
2. Suppose f is an open immersion with dense image, h is universally closed, and g is separated. Show that f is an isomorphism.
3. Suppose f is dominant, h is proper and g is separated. Show that g is proper.

Exercise 4

Show that properness is stable under composition and base change.