# Exercise sheet 16

Algebraic Geometry II
Summer term 2018

**Exercise 1**

Let $A$ be a ring and equip the polynomial rings $A[T_0, \dots, T_n]$ and $A[T_0, \dots, T_{n+1}]$ with their usual gradings. On the level of graded rings, the inclusion

$$i : A[T_0, \dots, T_n] \to A[T_0, \dots, T_{n+1}]$$

has a section $\pi$ defined by sending $T_{n+1}$ to 0. Using Proj, to what extent can you translate this situation into the world of schemes?

**Exercise 2**

Let $R$ be a ring and $A = R[T_0, \dots, T_n]$ be equipped with the usual grading. Calculate the global sections of $\mathrm{Proj}(A)$.

**Exercise 3**

Give an example of a closed immersion $i : Z \to X$ of schemes such that the induced morphism

$$i^{\#}(X) : \mathcal{O}(X) \to \mathcal{O}(Z)$$

is not surjective. Then give an example of a morphism $f : Z \to X$ where $f^{\#}(X)$ is surjective, but $f$ is not a closed immersion.

**Exercise 4**

Give an example of a graded ring $A$ such that the canonical morphism $\mathrm{Proj}(A) \to \mathrm{Spec}(A_0)$ is not quasi-compact.

**Exercise 5**

Let $k$ be a field, and $I = \langle x^2 - y^2 - x, x^2 - y^2 - y \rangle \in k[x, y]$ and ideal with corresponding closed subscheme

$$i : \mathrm{Spec}(k[x, y]/I) \to \mathrm{Spec}(k[x, y]) = \mathbb{A}^2_k.$$

Consider the standard embedding $j : \mathbb{A}^2_k \to \mathbb{P}^2_k$ and calculate carefully the scheme theoretic closure of $j \circ i$.

**Exercise 6**

Let $A$ be a non-negatively graded ring and $S$ an $A_0$-algebra. Show that $A \otimes_{A_0} S$ has an induced grading and construct an isomorphism

$$\mathrm{Proj}(A \otimes_{A_0} S) \cong \mathrm{Proj}(A) \times_{\mathrm{Spec}(A_0)} \mathrm{Spec}(S)$$

of schemes over $\mathrm{Spec}(A_0)$.