Exercise sheet 15

Algebraic Geometry II
Summer term 2018

Exercise 1

Let A be a graded ring and I a homogeneous ideal of A. Show that I is prime if and only if for all *homogeneous* $a, b \in A$ with $a \cdot b \in I$ one has $a \in I$ or $b \in I$.

Exercise 2

Let A be a non-negatively graded ring and $\mathfrak{p}$ be a homogeneous prime ideal. Show that $\mathcal{V}_+(\mathfrak{p})$ is an irreducible subset of $\mathrm{Proj}(A)$.

Exercise 3

Let A be a non-negatively graded ring and let d be a non-negative integer. Show that

$$B := A_0 \oplus \bigoplus_{k \geq d} A_k$$

is a graded subring of A. Show that intersection with B gives a bijection of sets $\mathrm{Proj}(A) \xrightarrow{\sim} \mathrm{Proj}(B)$. Are the two schemes $\mathrm{Proj}(B)$ and $\mathrm{Proj}(A)$ also isomorphic?

Exercise 4

Give an example of two graded rings A, B and a morphism $\mathrm{Proj}(B) \to \mathrm{Proj}(A)$ which is not induced from a graded ringhomomorphism $A \to B$.

Exercise 5

Let A be a ring and equip the polynomial rings $A[T_0, \ldots, T_n]$ and $A[T_0, \ldots, T_{n+1}]$ with their usual graduations. On the level of graded rings, the inclusion

$$i : A[T_0, \ldots, T_n] \to A[T_0, \ldots, T_{n+1}]$$

has a section defined by sending T_{n+1} to 0. Using Proj, to what extent can you translate this situation into the world of schemes?