Exercise sheet 9

Algebraic Geometry I
Winter term 2017/2018

Exercise 1

Let k be a field. Describe the scheme

$$\mathrm{Spec}(\mathrm{Quot}(k[X])) \times_{\mathrm{Spec}(k)} \mathrm{Spec}(\mathrm{Quot}(k[Y])).$$

Exercise 2

Consider a scheme S, a scheme $X \in \mathrm{Sch}/S$ and the fibre product $X \times_S X$. The universal property induces a morphism

$$\Delta \colon X \to X \times_S X$$

of schemes. On topological spaces, this yields a subset

$$\Delta(X) \subseteq \{y \in X \times_S X \mid pr_1(y) = pr_2(y)\}.$$

Show that this need not be an equality.
Hint: A candidate for X can be obtained by glueing two affine lines $\mathbb{A}^1$ along the open subsets $\mathbb{A}^1 \setminus \{0\}$.