Exercise sheet 8

Algebraic Geometry I
Winter term 2017/2018

Exercise 1

Consider a prime $p \in \mathbb{Z}$ and the scheme given as the restriction of the scheme $\mathbb{A}^1_{\mathbb{Z}} = \mathrm{Spec}(\mathbb{Z}[X])$ to the open set $U := \mathbb{A}^1_{\mathbb{Z}} \setminus (X, p)$. Show that this is not an affine scheme.

Exercise 2

Consider a prime $p \in \mathbb{Z}$ and $n \geq 1$ an integer. Describe the topological space and the structure sheaf of the scheme $\mathrm{Spec}(\mathbb{Z}/p^n)$.

Exercise 3

Let $F\colon X \to Y$ be a continuous map and let $\mathcal{G}$ be a sheaf on Y. Show that the presheaf

$$\begin{array}{rccl} f^+\mathcal{G}\colon & \mathrm{Ouv}(X)^{op} & \to & \mathcal{S} \\ & U & \mapsto & \operatorname*{colim}_{f(U) \subseteq V \in \mathrm{Ouv}(Y)^{op}} \mathcal{G}(V) \end{array}$$

is not a sheaf in general (with other words: construct a counterexample).

Exercise 4

Let $f\colon A \to B$ a ring homomorphism and consider the induced morphism $(F, F^\sharp)\colon \mathrm{Spec}(B) \to \mathrm{Spec}(A)$ of schemes. Show that

$$f \text{ is injective} \iff F^\sharp \text{ is stalkwise injective}$$

and

$$f \text{ is surjective} \iff \begin{array}{l} F \text{ is a closed embedding and} \\ F^\sharp \text{ is stalkwise surjective.} \end{array}$$