

Exercise sheet 6

Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1

Consider two points $p, q \in S^1$ with $p \neq q$ and the presheaf

$$\begin{aligned} \mathcal{F}: \text{Ouv}(S^1)^{op} &\rightarrow \mathbf{Set} \\ U &\mapsto \{f \in \mathcal{C}(U, \mathbb{R}) \mid f(p) = f(q) \text{ if } p, q \in U\}. \end{aligned}$$

Show that it is not a sheaf and determine its sheafification.

EXERCISE 2

Find counterexamples to the statements of Lemma 7.11, Lemma 7.12, and Lemma 7.13 in the case of \mathcal{F} and \mathcal{G} being just presheaves.

EXERCISE 3

Let $X \subseteq \mathbb{C}$ be an open non-empty subset. Show that

$$\begin{aligned} \mathcal{C}_{hol}(-, \mathbb{C}): \text{Ouv}(X)^{op} &\rightarrow \mathbf{Set} \\ U &\mapsto \{f \in \mathcal{C}(U, \mathbb{C}) \mid f \text{ is holomorphic}\}. \end{aligned}$$

is a sheaf. Show that the derivative $f \mapsto f'$ of a function $f: U \rightarrow \mathbb{C}$ induces a morphism $D: \mathcal{C}_{hol}(-, \mathbb{C}) \rightarrow \mathcal{C}_{hol}(-, \mathbb{C})$ of presheaves which is stalkwise surjective but not sectionswise surjective.

EXERCISE 4

Determine the sheafification of the presheaf $\mathcal{C}_{bd}(-, \mathbb{R})$ from the last exercise sheet.