Exercise sheet 5

Algebraic Geometry I
Winter term 2017/2018

Exercise 1

Let X be a topological space and $\mathcal{F}\colon \mathrm{Ouv}(X)^{op} \to \mathbf{Set}$ a sheaf. Let $s, t \in \mathcal{F}(X)$ and show that

$$Y := \{x \in X \mid s_x = t_x\}$$

is an open subset of X.

Exercise 2

Consider the presheaf

$$\begin{array}{rccl} \mathcal{C}_{bd}(-,\mathbb{R})\colon & \mathrm{Ouv}(\mathbb{R})^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \{f\colon U \to \mathbb{R} \mid \ f \text{ is continuous and bounded}\} \end{array}$$

and show that it is not a sheaf.

Exercise 3

Let $\pi\colon \tilde{X} \to X$ be a continuous map of topological spaces. Show that the presheaf

$$\begin{array}{rccl} \mathcal{S}ec(\pi)\colon & \mathrm{Ouv}(X)^{op} & \to & \mathbf{Set} \\ & U & \mapsto & \{s\colon U \to \tilde{X} \mid \pi s = \mathrm{id}_U \text{ and } s \text{ is continuous}\} \end{array}$$

is a sheaf. Consider the covering space $\pi\colon \mathbb{R} \to S^1$ given by the exponential function and draw a picture. Show that $\mathcal{S}ec(\pi)$ is not a constant sheaf (i.e. not the sheafification of a constant presheaf).

Exercise 4

Let $f, g\colon \mathcal{F} \to \mathcal{G}$ be maps of presheaves from a presheaf $\mathcal{F}$ into a sheaf $\mathcal{G}$ which coincide on each stalk, i.e. $f_x = g_x$ for all $x \in X$. Show that $f = g$.