Exercise sheet 4

Algebraic Geometry I
Winter term 2017/2018

Exercise 1

Consider the topological space $X := \mathrm{Spec}(A)$ for a ring A.

1. Let $\{Z_\alpha\}_\alpha$ be a collection of closed subsets $Z_\alpha \subseteq X$ such that the intersection of each finite subcollection is non-empty. Show that the whole intersection $\cap_\alpha Z_\alpha$ is non-empty.

2. Show exercise (a) in the proof of Corollary 5.10 of the script.

Exercise 2

Decide whether the canonical morphism $\coprod_{\mathbb{N}} \mathrm{Spec}(\mathbb{F}_2) \to \mathrm{Spec}(\prod_{\mathbb{N}} \mathbb{F}_2)$ is a homeomorphism in the Zariski topology.

Exercise 3

Show that $\mathbb{C}^2$ is not homeomorphic to $\mathbb{C}^1 \times \mathbb{C}^1$ in the Zariski Topology.

Exercise 4

Let k be a field.

1. Show that $\mathrm{Spec}(k[X_1, \dots, X_n])$ is infinite if $n \geq 1$.

2. Let A be a k-algebra of finite type with $\mathrm{Spec}(A)$ finite. Conclude that A is finite over k. (You may use that $\mathrm{Spec}(f)$ is surjective if f is injective and integral.)