Exercise sheet 3

Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1

Let A be a ring. Show that an element $f \in A$ is a unit if and only if $f(\mathfrak{p}) \neq 0$ for all $\mathfrak{p} \in \operatorname{Spec}(A)$.

EXERCISE 2

Let k be a field and consider the image $Y \subseteq \mathbb{A}^3$ of the algebraic map

$$\begin{array}{cccc} F\colon & \mathbb{A}^1 & \to & \mathbb{A}^3 \\ & t & \mapsto & (t, t^2, t^3). \end{array}$$

More precisely, one considers the associated morphism

$$\begin{array}{cccc} f\colon & k[X,Y,Z] & \to & k[T] \\ & X & \mapsto & t \\ & Y & \mapsto & t^2 \\ & Z & \mapsto & t^3 \end{array}$$

of k-algebras and sets $Y := \operatorname{Spec}(k[X,Y,Z]/\operatorname{Ker}(f)) \cong \mathcal{V}(\operatorname{Ker}(f)) \subseteq \mathbb{A}^3$. Find generators of the ideal $\operatorname{Ker}(f)$ and show that $Y \cong \mathbb{A}^1$ as topolgical spaces.

EXERCISE 3

Show that $\mathcal{V}(X^2 - YZ, XZ - X) \subseteq \mathbb{A}^3$ is a union of three irreducible components, each of them homeomorphic to $\mathbb{A}^1$.

EXERCISE 4

Describe the closed subsets of $\mathbb{A}^1_{\mathbb{C}}$.