Exercise sheet 2

Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1

Let A be a ring. Discuss the construction of the ring $A[\![X]\!]$ of *formal power series* and characterize its invertible elements. Describe $\operatorname{Spec}(k[\![X]\!])$ for a field k.

EXERCISE 2

Let X be a compact Hausdorff space and denote

$$\mathcal{C}(X,\mathbb{R}) := \{f\colon X \to \mathbb{R} \mid f \text{ is continuous}\}.$$

This is a ring by adding and multiplying value-wise. Consider the map

$$\begin{array}{rcl} \mu\colon X & \to & \operatorname{mSpec}(\mathcal{C}(X,\mathbb{R})) \\ x & \mapsto & \mathfrak{m}_x := \{f \in \mathcal{C}(X,\mathbb{R}) \mid f(x) = 0\}. \end{array}$$

and show that this is a homeomorphism where $\operatorname{mSpec}(\mathcal{C}(X,\mathbb{R}))$ carries the subspace topology of the Zariski topology on $\operatorname{Spec}(\mathcal{C}(X,\mathbb{R}))$.

EXERCISE 3

Find the radical ideal of (X^3Y^2).

EXERCISE 4

Find the irreducible components of $V((X^3 - YZ)^2, (XZ - Y^2)^3) \subseteq \mathbb{A}^3_{\mathbb{C}}$.