

Exercise sheet 12

Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1

Consider a morphism $f: X \rightarrow Y$ of schemes. Show that there is a unique closed subscheme $i: Z \hookrightarrow Y$, called the *scheme theoretic image of f* , with the following universal property:

1. There is a factorization $X \rightarrow Z \xrightarrow{i} Y$ of f .
2. For every other such factorization $X \rightarrow Z' \xrightarrow{i'} Y$ of f with i' a closed immersion, there exists a unique map $t: Z \rightarrow Z'$ making everything commute.

Show moreover that, if X was a reduced scheme, Z is the closure of the image $\overline{f(X)} \subseteq Y$ together with the reduced closed subscheme structure. (Hint: Consider the case of an affine Y first.)

EXERCISE 2

Show that there cannot be an equivalence of categories between the category **Set** of sets and its opposite category **Set**^{op}. (Hint: Use coproducts, products, final and initial objects.)