

# Exercise sheet 11

Algebraic Geometry I  
Winter term 2017/2018

## EXERCISE 1

Let  $k$  be a field and consider the scheme  $X$  glued out of  $X_1 := \text{Spec}(k[X])$  and  $X_2 := \text{Spec}(k[Y])$  along

$$\begin{array}{ccc} X_1 \setminus \{0\} \cong \text{Spec}(k[X, X^{-1}]) & \xrightarrow{\cong} & \text{Spec}(k[Y, Y^{-1}]) \cong X_2 \setminus \{0\} \\ X & \mapsto & Y. \end{array}$$

Show that  $X$  is not affine, that  $X$  is noetherian and that  $X$  is irreducible.

## EXERCISE 2

Find an open subscheme  $U$  of a quasicompact scheme  $X$  which is not quasicompact. Show that there is no such example if  $X$  is noetherian.

## EXERCISE 3

A sheaf  $\mathcal{F}$  on a topological space  $X$  is called *flasque*, if for every inclusion  $V \subseteq U$  of open subsets of  $X$ , the restriction map  $\mathcal{F}(U) \rightarrow \mathcal{F}(V)$  is surjective. Show that a constant sheaf on an irreducible space is flasque.

## EXERCISE 4

Show that an exact sequence

$$0 \rightarrow \mathcal{F}' \rightarrow \mathcal{F} \rightarrow \mathcal{F}'' \rightarrow 0$$

of sheaves of abelian groups such that  $\mathcal{F}'$  is flasque is exact (i.e. stalkwise exact) if and only if it is sectionwise exact.