Exercise sheet 10

Algebraic Geometry I
Winter term 2017/2018

For the first two exercises, let $k = \bar{k}$ be an algebraically closed field and consider the morphism

$$f\colon X := \operatorname{Spec}(k[X,Y]/(X - Y^2)) \to S := \operatorname{Spec}(k[T])$$

of schemes induced by $T \mapsto X$ (draw a picture!).

Exercise 1

Calculate the fibers X_s of f at all points $s := (T - a) \in S$ for $a \in k$.

Exercise 2

Calculate the fiber X_η of f at the generic point $\eta \in S$.

Exercise 3

Fix positive integers a, b and c and consider the Fermat scheme

$$S := \operatorname{Spec}(\mathbb{Z}[X,Y,Z]/(X^a + Y^b - Z^c))$$

with open subscheme $U := S \setminus \mathcal{V}((X,Y,Z) + (X^a + Y^b - Z^c))$. Show that the set of morphisms

$$\operatorname{Hom}_{\mathrm{Sch}}(\operatorname{Spec}(\mathbb{Z}), U)$$

is in bijection with the integer solutions (x,y,z) to $X^a + Y^b = Z^c$ with $ggT(x,y,z) = 1$.