

Exercise sheet 1

Algebraic Geometry I
Winter term 2017/2018

EXERCISE 1

Let A be a noetherian ring and $\mathfrak{p}' \subsetneq \mathfrak{p} \subsetneq \mathfrak{p}''$ a chain of prime ideals. Show that there are infinitely many prime ideals \mathfrak{p} with this property. (Hint: Use prime avoidance and Krull's height theorem.)

EXERCISE 2

Show that a ring A is reduced if and only if all localizations $A_{\mathfrak{p}}$ at prime ideals \mathfrak{p} are reduced. Show that the analogue is not necessarily true for irreducibility.

EXERCISE 3

Let A be a ring and consider the A -algebra homomorphism

$$\begin{aligned} \Psi': A[X_1, \dots, X_n] &\longrightarrow \text{Hom}_{\text{Set}}(A^n, A) \\ f &\mapsto (\underline{a} \mapsto f(\underline{a})), \end{aligned}$$

where the A -algebra structure on the set $\text{Hom}_{\text{Set}}(A^n, A)$ is given pointwise from the target A . Show that there exists a ring A such that Ψ' is not injective. Show that Ψ' is injective, if A is an infinite domain.

EXERCISE 4

Let k be a field and consider the k -subalgebra

$$A := \{f \in k[X] \mid f(0) = f(1)\}$$

of $k[X]$. Find a representation $A \cong k[Y, Z]/I$ for some ideal I . This shows that A is a k -Algebra of finite type.